November 2011

Electroweak scale invariant models with small cosmological constant

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Abstract

Scale invariant alternatives of the Standard Model are well motivated as they solve the hierarchy problem in a technically natural way. In this paper we consider a class of phenomenologically consistent scale invariant models where the scale invariance is broken radiatively at the electroweak scale. We show that the simplest such electroweak scale invariant model, which contains an electroweak Higgs doublet and an extra singlet scalar field, has the same number of parameters and hence same predictive power as the Standard Model. We argue that the vanishing of the cosmological constant in such models constrains the mass of the pseudo-Goldstone boson associated with this breaking to be less than ~ 10 GeV and also leads to a fermionic-bosonic mass relation. We also consider extended scale invariant models that incorporate neutrino masses through couplings to an electroweak triplet scalar field and dark matter within a mirror-symmetric framework. Scale invariant theories as developed here are highly predictive and can be probed at the LHC.

1 Introduction

The quantum stability of the electroweak scale is one of the main motivations for new physics as it suggests enlarged symmetry of the Standard Model. In this regard, scale invariance is an interesting candidate for such an additional symmetry of particle interactions. Scale invariance, can be an exact classical symmetry, broken radiatively as a result of the quantum anomaly. This generates a mass scale through the quantum-mechanical phenomenon of dimensional transmutation [1]. Despite its anomalous nature, scale invariance still ensures the stability of the electroweak scale since the radiative breaking is "soft" as it generates only logarithmic corrections to the electroweak scale. Realistic scale invariant models broken perturbatively by quantum corrections can be constructed which typically feature a scalar sector consisting of the usual Higgs doublet together with a Higgs singlet and possibly other scalar fields [2, 3, 4, 5]. For related works see also [6, 7, 8, 9, 10, 11, 12].

It is possible that all scales: electroweak, neutrino mass, cosmological constant and Planck scales originate radiatively via quantum corrections. In this article, we will define a class of scale invariant theories which we call 'electroweak scale invariant models' whereby the 'low energy' effective Lagrangian describing physics below the Planck scale is scale invariant. That is, the relevant particle physics scales such as the electroweak scale and the neutrino mass scale are generated through the dimensional transmutation at energies $\sim$TeV.

Quantum corrections in scale invariant models also generate a finite, and thus in principle calculable, cosmological constant (CC). Constraining the cosmological constant to be small imposes interesting constraints on scale invariant models [13]. We first set aside the issue of neutrino mass and reconsider the simplest, phenomenologically consistent, scale invariant electroweak model which features a scalar content consisting of a Higgs doublet and a real singlet [2]. Scale invariance together with the small CC leads to a highly predictive theory with the same number of parameters as the standard model. This theory has a leading order prediction for the Higgs mass of around $280 - 305$ GeV and also contains a light PGB with mass less than about 10 GeV. It turns out this minimal scale invariant model is still consistent with precision electroweak data, because of the effects of the light PGB. The model is also marginally consistent with the recent LHC data [14] on the direct Higgs boson search. We also discuss two extensions of the minimal model. One incorporates neutrino masses via the type II see-saw mechanism utilizing a scalar triplet $\Delta \sim (1, 3, 2)$, and another introduces dark matter within a mirror-symmetric hidden sector model [15].

2 Perturbatively small cosmological constant in scale invariant theories

The incorporation of a small cosmological constant within scale invariant theories has been discussed in Ref. [13]. It will lead to important constraints on realistic scale invariant theories, so we here briefly review this material.

Consider a classically scale-invariant theory that contains a set of n real scalar fields S_i

$(i = 1, 2, ...n)$. Some of these scalar fields may form multiplets of a local or global symmetry group. The generic classical potential can be written as [16]:

$$V_0(S_i) = \lambda_{ijkl} S_i S_j S_k S_l \ , \tag{1}$$

where λ_{ijkl} are bare coupling constants and summation over the repeated indices is assumed. It is convenient to adopt the hyper-spherical parameterization for the scalar fields:

$$\begin{aligned} S_i(x) &= r(x) \cos\theta_i(x) \prod_{k=1}^{i-1} \sin\theta_k(x) \ , \ \text{for } i = 1, \ldots, n-1 \\ S_n(x) &= r(x) \prod_{k=1}^{n-1} \sin\theta_k \end{aligned} \tag{2}$$

where $r(x)$ is the modulus field. Its nonzero VEV, $\langle r \rangle \neq 0$, breaks scale invariance spontaneously resulting in a corresponding (pseudo-)Goldstone boson, the dilaton [16], [1]. In the parameterization of (2) the classical potential takes the form

$$V_0(r, \theta_i) = r^4 f(\lambda_{ijkl}, \theta_i) \ . \tag{3}$$

Due to the classical scale invariance the modulus field $r(x)$ factors out, and the extremum condition $\frac{\partial V_0}{\partial r}\big|_{r=\langle r \rangle,\ \theta_i=\langle \theta_i \rangle} = 0$ implies that the VEV of the potential, that is, the classical contribution to the CC, vanishes: $V_0(\langle r \rangle, \langle \theta_i \rangle) = 0$. At the classical level, the dilaton field remains massless and the VEV $\langle r \rangle = 0$, unless $f(\lambda_{ijkl}, \theta_i) = 0$ in which case $\langle r \rangle$ is undetermined (flat direction).

Quantum corrections lead to an effective potential which can be written in terms of effective, renormalization scale μ-dependent couplings and fields,

$$\begin{aligned} V = A(g_a(\mu), m_x(\mu), \theta_i(\mu), \mu) r^4(\mu) + B(g_a(\mu), m_x(\mu), \theta_i(\mu), \mu) r^4(\mu) \log\left(\frac{r^2(\mu)}{\mu^2}\right) \\ + C(g_a(\mu), m_x(\mu), \theta_i(\mu), \mu) r^4(\mu) \left[\log\left(\frac{r^2(\mu)}{\mu^2}\right)\right]^2 + \ldots \ , \end{aligned} \tag{4}$$

where $\ldots$ denotes all terms with higher-power logarithms coming from all possible higher-loop diagrams. The parameters $g_a(\mu)$ and $m_x(\mu)$ denote all relevant running dimensionless couplings and effective masses, respectively. For our purposes it is very convenient to fix the renormalization scale as $\mu = \langle r \rangle$. With this choice of μ the higher-power log terms become irrelevant for our discussion and we do not need to display them here. In addition, since we are primarily interested in the VEV of the effective potential (4) we fix the direction of the potential by taking $\theta_i = \langle \theta_i \rangle$ in (4). The extremum condition along the radial direction implies

$$\frac{\partial V}{\partial r} = 0 \quad \Rightarrow \quad 2A(\mu = \langle r \rangle) + B(\mu = \langle r \rangle) = 0 \ . \tag{5}$$

If we demand that the perturbative contribution to the CC vanishes, then this requires that $V_{\rm min} = 0$, that is,

$$V_{\rm min} = 0 \quad \Rightarrow \quad A(\mu = \langle r \rangle) = 0 \ . \tag{6}$$

Note that while $V_{\rm min} = 0$ implies tuning of parameters, the condition (5) is just an extremum condition which simply implies that the scale $\langle r \rangle$ is defined as the scale μ where $2A + B = 0$. Thus, the condition (5) trades one dimensionless parameter for a dimensional parameter, the phenomenon known as dimensional transmutation.

Evidently, with the above conditions (5) and (6) the mass of the dilaton $m_{\rm PGB} = \left.\frac{\partial^2 V}{\partial r^2}\right|_{r=\mu=\langle r\rangle, \langle \theta_i \rangle}$ is determined by at least two-loop level quantum corrections,

$$m^2_{\rm PGB} = 8C(\mu = \langle r \rangle)\langle r \rangle^2 \ . \tag{7}$$

Clearly, we must require $C(\mu = \langle r \rangle) > 0$ for the fine tuning of Eq. (6) to be acceptable.

The renormalization group (RG) properties of the effective potential (4) give further relations between A, B and C. The potential should not depend on the renormalization scale μ, that is,

$$\mu \frac{dV}{d\mu} \equiv \left(\mu \frac{\partial}{\partial \mu} + \sum_a \beta_a \frac{\partial}{\partial g_a} + \sum_x \gamma_x m_x \frac{\partial}{\partial m_x} - \gamma_r r \frac{\partial}{\partial r} - \sum_i \gamma_i \theta_i \frac{\partial}{\partial \theta_i} \right) V = 0 \ , \tag{8}$$

where β_a are beta-functions which determine the running of couplings g_a, while γ_r, γ_i are scalar anomalous dimensions and $\gamma_x \equiv \frac{\mu}{m_x}\frac{\partial m_x}{\partial \mu}$ are mass anomalous dimensions. Equations (5), (6) and (8) imply

$$\begin{aligned} B(\mu = \langle r \rangle) &= \left. \frac{1}{2}\mu \frac{dA}{d\mu} \right|_{\mu=\langle r \rangle} , \\ C(\mu = \langle r \rangle) &= \left. \frac{1}{4}\mu \frac{dB}{d\mu} \right|_{\mu=\langle r \rangle} . \end{aligned} \tag{9}$$

The quantities A, B and C can in principle be computed in perturbation theory. The leading-order contributions to A, B and C arise at tree "(0)", one loop "(1)", and two loops "(2)", respectively, and if perturbation theory is valid, then the conditions $A(\mu = \langle r \rangle) = 0$, $B(\mu = \langle r \rangle) = 0$ and $C(\mu = \langle r \rangle) > 0$ imply:

$$\begin{aligned} &A^{(0)}(\mu = \langle r \rangle) \approx 0 \ , \\ &B^{(1)}(\mu = \langle r \rangle) \approx 0 \ , \\ &C^{(2)}(\mu = \langle r \rangle) > 0 \ . \end{aligned} \tag{10}$$

The first condition can be used simply to define approximately the scale $\mu = \langle r \rangle$, and results in the elimination of one of the tree-level parameters in the potential. The quantity $B^{(1)}$ is in general

$$B^{(1)}(\mu = \langle r \rangle) = \left. \frac{1}{64\pi^2 \langle r \rangle^4}[3{\rm Tr}m_V^4 + {\rm Tr}m_S^4 - 4{\rm Tr}m_F^4] \right|_{\mu=\langle r \rangle} , \tag{11}$$

where the subscripts V, S and F denote contributions of massive vector bosons, scalars and Dirac fermions, respectively. The quantity $C^{(2)}$ then is given by

$$C^{(2)}(\mu = \langle r \rangle) = \frac{1}{64\pi^2 \langle r \rangle^4} \left[3\mathrm{Tr} m_V^4 \gamma_V + \mathrm{Tr} m_S^4 \gamma_S - 4\mathrm{Tr} m_F^4 \gamma_F \right] \Big|_{\mu = \langle r \rangle} , \tag{12}$$

where we have used (9). *A priori*, $C^{(2)}$ in (12) is not positive, thus the condition $C^{(2)} > 0$ puts a restriction on the particle spectrum of the theory. The condition $B^{(1)} \approx 0$ leads to the fermion-boson mass relation:

$$\left(3\mathrm{Tr} m_V^4 + \mathrm{Tr} m_S^4 - 4\mathrm{Tr} m_F^4 \right) |_{\mu = \langle r \rangle} \approx 0 . \tag{13}$$

Since the above constraints play a crucial role in the scale invariant models to be discussed below, and the CC is a relevant observable only in the presence of gravity, we would like to briefly mention how gravity can be incorporated within the given framework. A simple way is to assume that the Planck mass is also generated radiatively through the couplings $\sqrt{-g}\xi_{ij} S_i S_j R$, where R is the Ricci scalar. Because we assume that scale invariance is broken at the electroweak scale, to explain the weakness of the gravitational force one needs to introduce hierarchically large ξ_{ij} parameters or, alternatively, one can assume a large number of scalar fields. One may also assume that the Planck mass is spontaneously or dynamically generated in some hidden sector which extremely weakly interacts with the low-energy fields, rendering the effective low-energy theory to be approximately scale invariant. More detailed discussion of these possibilities is beyond the scope of the present paper.

3 The minimal scale invariant model revisited

Let us now apply the formalism outlined in the previous section to the simple Higgs doublet (ϕ) and one real singlet (S) scale invariant model [2]. The most general scale invariant potential is:

$$V_0(S_1, S_2) = \frac{\lambda_1}{2} \phi^\dagger \phi \phi^\dagger \phi + \frac{\lambda_2}{8} S^4 + \frac{\lambda_3}{2} \phi^\dagger \phi S^2 . \tag{14}$$

We parameterize the fields in unitary gauge through

$$\phi = \frac{r}{\sqrt{2}} \begin{pmatrix} 0 \\ \sin\theta \end{pmatrix}, \; S = r\cos\theta , \tag{15}$$

and we choose the $\lambda_3 < 0$ parameter space. In this case, $V_0(r) = A^{(0)} r^4$ and $A^0(\mu = \langle r \rangle) = 0$ implies

$$\sqrt{2}\langle \phi \rangle = \langle r \rangle \left(\frac{1}{1+\epsilon} \right)^{1/2} \equiv v \simeq 246 \; GeV, \; \langle S \rangle = v\epsilon^{1/2}, \tag{16}$$

where

$$\langle\theta\rangle = \omega\ , \quad \cot^2\omega \equiv \epsilon = \sqrt{\frac{\lambda_1(\mu)}{\lambda_2(\mu)}}\ , \tag{17}$$

with

$$\lambda_3(\mu) + \sqrt{\lambda_1(\mu)\lambda_2(\mu)} = 0 \tag{18}$$

and $\mu = \langle r\rangle$.

The model has two physical scalars, but only one of them gains mass at tree-level. The other scalar is the PGB, which gains mass at two loop level. The tree-level mass can easily be obtained from the tree-level potential, by defining shifted fields: $\phi = \langle\phi\rangle + \phi',\ S = \langle S\rangle + S'$. We find:

$$m_H^2 = \lambda_1 v^2 - \lambda_3 v^2,\ H = \cos\omega\phi_0' - \sin\omega S', \tag{19}$$

while the PGB is $h = \sin\omega\phi_0' + \cos\omega S'$.

The mass of the PGB can be calculated from Eq.(7,12), and is expected to be quite small (less than 10 GeV) since it arises at two loop level. Such a light PGB can only be possible if its coupling to standard model fields is suppressed. Thus the only phenomenologically consistent parameter space is where $\tan\omega < 1$ so that the PGB is mainly singlet. In fact, constraints from LEP require $\sin\omega \lesssim 0.3$ [17].

As discussed in the previous section, the incorporation of a small CC into scale invariant theories implies some constraints on parameters. The main constraint is that $B(\mu = \langle r\rangle) = 0$. To leading order in perturbation theory, this implies that $B^{(1)} \approx 0$ which leads to the contraint:

$$m_H^4 \approx 12 m_t^4\ . \tag{20}$$

Note that the above relation is evaluated for the running masses at a scale $\mu = \langle r\rangle$. Because of the smallness of $\sin\omega$, we can use the known Standard Model relations between the pole and running masses in our approximate calculations. Eq. (20) thereby implies that the physical Higgs mass M_H can be approximately related to the physical top quark mass M_t via:

$$M_H = (12)^{1/4}\left(1 - \frac{3}{4}\frac{\alpha_S(\langle r\rangle)}{\pi} - \delta_H(\langle r\rangle)\right) M_t\ , \tag{21}$$

where [18]

$$\delta_H\left(\langle r\rangle\right)) = \frac{G_F}{\sqrt{2}}\frac{m_H^2}{16\pi^2}\left[6\ln\left(\frac{\langle r\rangle^2}{m_H^2}\right) + \frac{9}{2}\left(\frac{25}{9} - \sqrt{\frac{1}{3}}\pi\right)\right]\ . \tag{22}$$

Taking $\alpha_s(M_Z) \approx 0.118$ and $M_t \approx 173$ GeV, we predict the Higgs pole mass $M_H \approx 280 - 305$ GeV for 1 TeV $\gtrsim \langle r\rangle \gtrsim 300$ GeV. For larger values of $\langle r\rangle$ the perturbative approximation begins to break down.

As indicated in Eqs.(7) and (12), the mass of the PGB depends on the anomalous mass dimensions for the scalar H and the top quark. Evaluating these anomalous mass dimensions in the relevant parameter regime where $\cos^2\omega \approx 1$, we find:

$$\begin{aligned}\gamma_S &= \frac{3\lambda_1}{4\pi^2} - \frac{9\lambda_D^2}{8\pi^2}\left(\frac{m_t^2}{m_H^2} - \frac{1}{6}\right) \\ \gamma_F &= \frac{3\lambda_D^2}{32\pi^2} - \frac{2\alpha_s}{\pi} \ .\end{aligned} \tag{23}$$

Using $\lambda_D^2 = 2m_t^2/v^2$ and the relation Eq.(20), we find

$$C^{(2)} = \frac{3m_t^4 \sin^4\omega}{16\pi^2 v^4}\left(\frac{2\alpha_s}{\pi} + \frac{3m_t^2}{v^2\pi^2}\left[\frac{3\sqrt{3}}{8} + \frac{1}{16}\right]\right) \ . \tag{24}$$

Evidently $C^{(2)} > 0$ and hence the model is consistent with the inferred small CC. The PGB mass can then be estimated from Eq.(7):

$$\begin{aligned} m_{\rm PGB} &\approx \sqrt{\frac{3}{2}}\frac{m_t^2}{\pi v}\sin\omega\left[\frac{m_t^2}{\pi^2 v^2}\left(\frac{9\sqrt{3}}{8} + \frac{3}{16}\right) + \frac{2\alpha_s}{\pi}\right]^{1/2} \\ &\approx 7\left(\frac{\sin\omega}{0.3}\right)\ GeV \ . \end{aligned} \tag{25}$$

For such a light PGB, LEP bounds limit $\sin\omega \lesssim 0.3$ [17].

If $\sin\omega$ is close to the experimental limit then the PGB can potentially be experimentally probed at the LHC. Firstly it can be directly produced via, e.g., $pp \to t\bar{t} + h$. Secondly, the PGB can also manifest itself indirectly through radiative corrections. In particular indirect limits on the standard model Higgs mass arise from oblique radiative corrections involving the Higgs contributions to the W, Z self energies. These contributions are proportional to $\ln\ M_{\rm Higgs}^2$, where $M_{\rm Higgs}$ is the Higgs boson mass within the Standard Model. Thus the effect of the PGB is to replace this dependence with

$$\ln\ M_{\rm Higgs}^2 \to \cos^2\omega \ln\ M_H^2 + \sin^2\omega \ln\ m_{\rm PGB}^2 \tag{26}$$

The 90% C.L. (99% C.L.) upper limit on the standard model Higgs mass from precision electroweak data is [19]

$$M_{\rm Higgs} < 145(194)\ {\rm GeV\ (Standard\ model)}\ . \tag{27}$$

Interestingly, the existence of a light PGB can weaken the bound on M_H c.f. standard model Higgs. Indeed, Eqs.(26) and (27) suggest a 90% (99%) C.L. limit on M_H of

$$M_H < 288(396)\ {\rm GeV} \tag{28}$$

where we have assumed a mixing angle at the LEP limit: $\sin\omega \approx 0.3$. The above limit from the precision electroweak data is compatible with the leading order prediction of $M_H \approx 280 - 305$ GeV.

The scalar H in the model interacts with the Standard Model particles similar to the Standard Model Higgs boson except with couplings reduced by $\cos\omega$. Note that the decay modes $H \to hh$, $H \to hhh$ are absent at tree-level since the corresponding interactions vanish as a result of scale invariance. Therefore, experimental limits on Standard Model Higgs can be easily adjusted to H. The very recent analysis of 1-2 fb^{-1} LHC data [14] shows that large regions of the Higgs boson mass below 460 GeV are already excluded at 95% C.L. A small region $288 - 296$ GeV is still allowed and the minimal model is currently consistent with experiments. However, at the same time, no significant excess is seen in the data within the mass region of interest either, and the minimal model seems to be excluded at 90% C.L. This is one of the motivations to consider extensions of the minimal model. Also, in a theory with the Higgs boson mass ~ 300 GeV, one expects the Landau pole for self-interaction coupling to be below the Planck mass. This theoretical issue does not necessarily undermine the minimal model considered above. However, it is desirable to extend the theory in such a way that removes the Landau pole. Further incentive for extending the minimal scale-invariant model is the necessity to incorporate neutrino masses and dark matter. In what follows we consider two such extensions of the minimal model which are motivated by the above issues.

4 Neutrino masses in scale-invariant models

The simplest way to generate Dirac neutrino masses is to extend the minimal model by introducing right-handed neutrinos which allow for tiny Yukawa couplings. One could also consider a model with type I see-saw where the right-handed Majorana neutrino masses are generated through the couplings with the singlet scalar field of the minimal model. Similarly, one could generate Majorana masses for triplet fermions and generate light neutrino masses via the type III see-saw mechanism. However, both type I and III models introduce new heavy fermionic states. This can lead to tension with precision electroweak data if these new fermionic states are heavier than the top-quark mass scale, because this potentially increases the scalar masses suggested by the mass relation Eq. (13) above the limits Eq. (28).

In contrast the type II see-saw mechanism introduces only new bosonic degrees of freedom and thus reduces the prediction of the Higgs mass compared with the minimal model. While type I and III see-saw models are fairly trivial modifications of the analyses of the previous section because they do not require any new scalars, type II is non-trivial in this respect. We now discuss the type II see-saw scale-invariant model in more detail.

We extend the minimal model of the previous section by introducing the electroweak triplet scalar field Δ [3]. Neutrino masses are generated from the Lagrangian term

$$\mathcal{L} = \lambda \bar{\ell}_L \Delta \tilde{\ell}_L + H.c.. \tag{29}$$

Here

$$\Delta \sim (1,3,-2) = \begin{pmatrix} \Delta^-/\sqrt{2} & \Delta^0 \\ \Delta^{--} & -\Delta^-/\sqrt{2} \end{pmatrix} \tag{30}$$

transforms like $\Delta \to U\Delta U^\dagger$ under $SU(2)_L$ and $\tilde{\ell_L} \equiv i\tau_2(\ell_L)^c \to U\tilde{\ell_L}$. A small VEV for the electrically neutral component Δ^0 generates a tree-level Majorana mass for ν_L.

The simplest phenomenologically-consistent scale-invariant potential which can give $\langle\Delta^0\rangle \neq 0$ requires ϕ, Δ and the real gauge singlet scalar field, S [3].[1] The most general tree-level potential is then

$$\begin{aligned} V_0 &= \lambda_1(\phi^\dagger\phi)^2 + \lambda_2(\mathrm{Tr}\Delta^\dagger\Delta)^2 + \lambda_2'\mathrm{Tr}(\Delta^\dagger\Delta\Delta^\dagger\Delta) + \frac{\lambda_3}{4}S^4 + \lambda_4\phi^\dagger\phi\mathrm{Tr}\Delta^\dagger\Delta + \lambda_4'\phi^\dagger\Delta\Delta^\dagger\phi \\ &+ \lambda_5\phi^\dagger\phi S^2 + \lambda_6\Delta^\dagger\Delta S^2 + \lambda_7\phi^\mathrm{T} i\tau_2\Delta\phi S + H.c. \end{aligned} \tag{31}$$

Note that only the λ_7 term violates lepton number. If in the limit $\lambda_7 \to 0$ the parameters are such that $\langle\phi_0\rangle = v$, $\langle S\rangle = w$, $\langle\Delta\rangle = 0$ then taking λ_7 small but nonzero will induce a VEV for the real part of the neutral component of Δ:

$$\langle\Delta_0\rangle = -\frac{\lambda_7 w}{\sqrt{2}(\lambda_4 + 2\lambda_6 w^2/v^2)}. \tag{32}$$

Minimising the tree-level potential in the limit $\lambda_7 \to 0$ leads to the relations,

$$\lambda_5(\Lambda) = -\sqrt{\lambda_1(\Lambda)\lambda_3(\Lambda)} \tag{33}$$

and

$$\frac{w^2}{v^2} = \sqrt{\frac{\lambda_1(\Lambda)}{\lambda_3(\Lambda)}}. \tag{34}$$

A small but nonzero λ_7 induces order λ_7^2 corrections to these formulas.

We can calculate the tree-level masses by expanding around the vacuum: $\phi = \langle\phi\rangle + \phi'$, $S = \langle S\rangle + S'$ and $\Delta = \langle\Delta\rangle + \Delta'$. Taking the limit $\lambda_7 \to 0$ we find that the physical scalar spectrum consists of an approximately degenerate complex Δ' triplet, a massive singlet $H = -\sin\theta\phi_0' + \cos\theta S'$, and a massless state $h = \cos\theta\phi_0' + \sin\theta S'$ (this is the PGB which will gain mass at two-loop level), where

$$\begin{aligned} \tan^2\theta &= \sqrt{\frac{\lambda_1}{\lambda_3}}, \\ m_\Delta^2 &= \frac{\lambda_4}{2}v^2 + \lambda_6 w^2, \\ m_H^2 &= 2\lambda_1 v^2 - 2\lambda_5 v^2. \end{aligned} \tag{35}$$

[1]The minimal scale-invariant Higgs potential containing only ϕ and Δ will preserve lepton number, and if $\langle\Delta\rangle \neq 0$ will lead to an experimentally excluded Majoron, as well as other light scalars.

Observe that there is no pseudo Goldstone boson associated with lepton number violation. This is because lepton number is explicitly broken and in the limit where the explicit lepton number violating term ($\lambda_7 \phi^{\mathrm{T}} i\tau_2 \Delta \phi S$) vanishes our parameter choice is such that lepton number is *not* spontaneously broken.

In this model, the incorporation of a small CC implies the constraint:

$$6m_\Delta^4 + m_H^4 \approx 12 m_t^4 \ . \tag{36}$$

Note the above relation holds for running masses evaluated at the scale $\mu = \langle r \rangle$. The Δ states are not expected to contribute to oblique radiative corrections, and, therefore, they can be heavier than m_H. In this case m_H can be light enough to avoid the Landau pole below the Planck mass. Thus, we end up with a prediction for m_Δ of :

$$m_\Delta \approx 190 \ GeV. \tag{37}$$

Importantly, this puts the mass of the Δ in the range where it can be probed at the LHC. Meanwhile the mass limit on H from precision electroweak data can be much weaker than in the SM [Eq.(27)] due to the presence of the light PGB and can be as weak as Eq.(28).

5 Mirror symmetric extension

Experimental bounds on the Higgs boson mass can be alleviated in generic hidden sector models where the Higgs can have a significant branching fraction to invisible decay modes. In this section we consider the case where the hidden sector is isomorphic to the ordinary sector. This allows an exact parity symmetry to be defined which interchanges the ordinary particles with their mirror counterparts, along with $x \to -x$. The mirror particles provide a candidate for the inferred dark matter in the Universe which can also explain the DAMA[20], CoGeNT[21] and CRESST-II [22] direct detection experiments [23].

The scalar content consists of ϕ, ϕ', S where S is a gauge singlet. Under mirror symmetry, $\phi \leftrightarrow \phi', \ S \to S$. The classical scale invariant potential is then:

$$V = \frac{\lambda_1}{2}(\phi^\dagger\phi\phi^\dagger\phi + \phi'^\dagger\phi'\phi'^\dagger\phi') + \frac{\lambda_2}{8}S^4 + \frac{\lambda_3}{2}S^2(\phi^\dagger\phi + \phi'^\dagger\phi') + \lambda_4\phi^\dagger\phi\phi'^\dagger\phi' \tag{38}$$

As before, quantum corrections can generate a non-trivial vacuum. We can parameterize the fields in unitary gauge through:

$$\phi = \frac{r}{\sqrt{2}}\begin{pmatrix} 0 \\ \sin\theta\cos\phi \end{pmatrix}, \ \phi' = \frac{r}{\sqrt{2}}\begin{pmatrix} 0 \\ \sin\theta\sin\phi \end{pmatrix}, \ S = r\cos\theta \ , \tag{39}$$

We consider the parameter space with $\lambda_1, \lambda_2 > 0$, $\lambda_3 < 0$ and $|\lambda_4| < \lambda_1$. With this parameter range mirror symmetry is unbroken.

In this case, $V_0(r) = A^{(0)} r^4$ and $A^0(\mu = \langle r \rangle) = 0$ implies

$$\langle \phi \rangle = \langle \phi' \rangle = (0, \frac{v}{\sqrt{2}})^T \ , \ \langle S \rangle = \sqrt{2} v \epsilon^{1/2}, \tag{40}$$

where

$$v \equiv \frac{\langle r \rangle}{\sqrt{2}} \left(\frac{1}{1+\epsilon} \right)^{1/2} \equiv v \simeq 246 \ GeV \ . \tag{41}$$

Also,

$$\langle \phi \rangle = \frac{\pi}{4}, \ \langle \theta \rangle = \omega \ , \quad \cot^2 \omega \equiv \epsilon = \sqrt{\frac{\lambda_1(\mu) + \lambda_4(\mu)}{2\lambda_2(\mu)}} \ , \tag{42}$$

with

$$\lambda_3(\mu) + \frac{1}{\sqrt{2}} \sqrt{\lambda_2(\mu)(\lambda_1(\mu) + \lambda_4(\mu))} = 0 \tag{43}$$

and $\mu = \langle r \rangle$.

At tree level we have a massless PGB together with two massive scalars:

$$\begin{aligned} m_{H_1}^2 &= (\lambda_1 + \lambda_4 - 2\lambda_3)v^2 \\ m_{H_2}^2 &= (\lambda_1 - \lambda_4)v^2 \ . \end{aligned} \tag{44}$$

Constraining the CC to be small suggests the mass relation:

$$m_{H_1}^4 + m_{H_2}^4 \simeq 24 m_t^4 \ . \tag{45}$$

This suggests an upper bound of around 360 GeV for $H_{1,2}$. A lower bound on $H_{1,2}$ comes from LEP experiments. If kinematically allowed, the $H_{1,2}$ can be produced via the channel $e^+e^- \to Z^* \to ZH_{1,2}$. The Higgs $H_{1,2}$ can either decay invisibly or to standard model particles. It turns out the former gives the strongest limit, which is $m_{H_{1,2}} > 104$ GeV at 95% C.L. [24]. As before, we expect the PGB to be light: $m_h \lesssim 10$ GeV. This model can easily accommodate the precision electroweak constraints and will be tested in the near future at the LHC. The results depend on the size of the mass splitting $|m_{H_2} - m_{H_1}|$ [25]. In particular, if the mass difference $|m_{H_2} - m_{H_1}|$ is greater than the experimental resolution then the standard Higgs search channels should eventually show two signals, each with cross section only 25% that of the standard model Higgs. [This assumes that the $H_1 - H_2$ mass difference is not so great as to allow a $H_2 \to H_1 H_1$ or $H_1 \to H_2 H_2$ to be kinematically allowed]. If $|m_{H_2} - m_{H_1}|$ is less than the experimental resolution but greater than the decay widths, then the standard Higgs search channels should show one signal with cross section 50% that of the Standard Model Higgs. In this case the mass relation Eq. (45) suggests $M_H \approx 300$ GeV.

6 Conclusion

The effective theory at energies below the Planck scale might well be scale invariant at the classical level, broken perturbatively only by quantum corrections. Such theories can be well

motivated as they can solve the hierarchy problem thus representing an interesting alternative to the standard model and its supersymmetric extension. We have constructed the simplest such theories whereby the electroweak, cosmological constant and the neutrino mass scale arise perturbatively via quantum corrections. Such electroweak scale invariant theories are highly predictive, the simplest example has the same number of parameters as the standard model. These theories generally lead to a bosonic - fermionic mass relation which can be tested experimentally. These scale invariant models also feature a PGB which is typically very light (less than 10 GeV) and can be probed at the LHC, either via direct production, or indirectly through oblique radiative corrections.

Acknowledgements

We would like to thank Ray Volkas for collaboration during the early stage of this work. The work was supported in part by the Australian Research Council.

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